✱ A **function** is a relation between a set of inputs & a set of **permissible** outputs.

↑
*(meaning every element in the input set has one & only one image in the output set)* ⟷ *In case you have $f(1)=2$ & $f(1)=7$ in a list of data, then $f$ is not a function.*
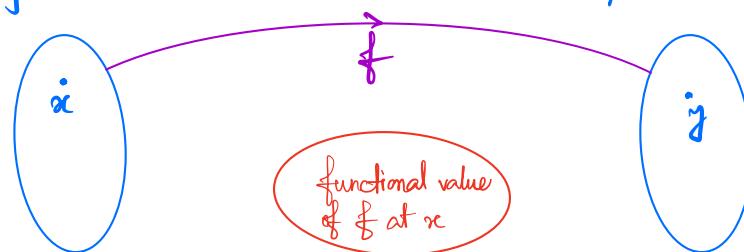
<u>Note</u>: We call functions as mappings too, as it maps one element of Input set to one-unique element in the Output set.
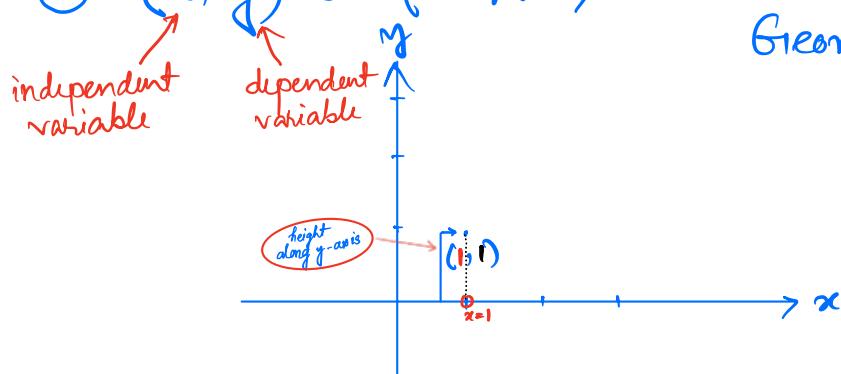
<u>Notation</u> :- ① Input Set     Output Set



*functional value of $f$ at $x$*

$y = f(x) \rightsquigarrow$ '$y$' is equal to '$f$' of '$x$'.

② $(x, y)$ or $(x, f(x)) \rightsquigarrow$ More used for Geometry.

*independent variable*    *dependent variable*

*height along y-axis* $\rightarrow (1, f)$

$x=1$

✱ The set of inputs is called the **Domain** & the set of outputs is called the **Range** of the function.

<u>Note</u> : There is something called ==Codomain== , it contains the Range of the function as a set.
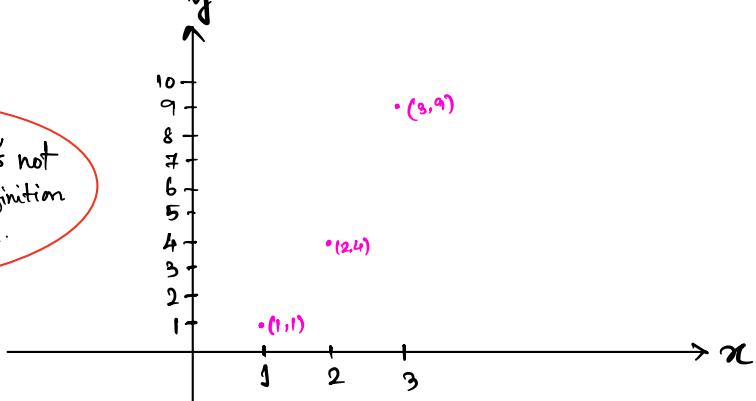


&#42; Graph of a Function : It's the geometry associated to the function.

Say, Domain = $\{1, 2, 3\}$ ↝ discrete, finite domain.

Codomain = the set of real numbers.

Range = $\{1, 4, 9\}$

In this notation it's not clear what's the definition or rule of the function.
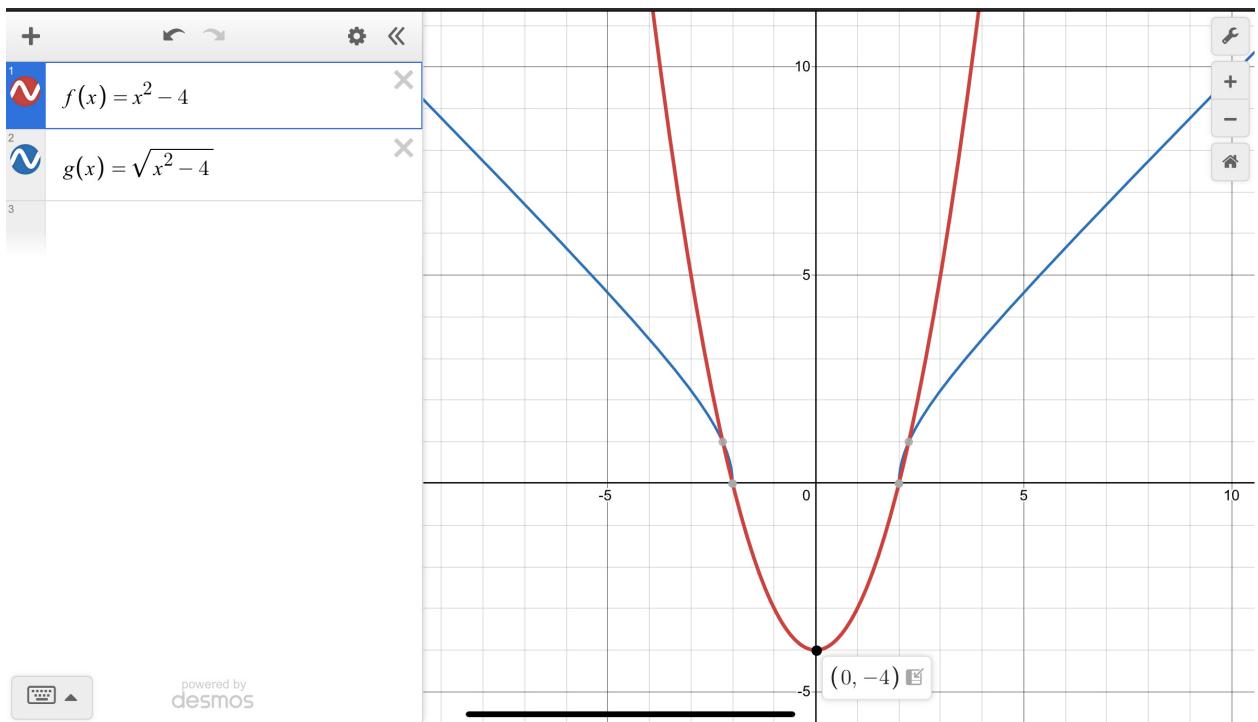


If we write $(1,1)$ , $(2,4)$ , $(3,9)$ , then it makes complete sense to draw, as we don't know the exact function.

Say, Domain = the set of real numbers. ≡ ℝ

Codomain = the set of real numbers.

function is defined as ① $f(x) = x^2 - 4$

② $g(x) = \sqrt{x^2 - 4}$



What we can say about range?

Range of function $f$ is all real numbers greater than or equal to $-4$. ($[-4, \infty)$ or $y \geq -4$)

Range of function $g$ is all real numbers greater than or equal to $0$. ($[0, \infty)$ or $y \geq 0$)

⊛ Set Builder Notation:- $\{\ x\ |\ x\ \text{has some property}\ \}$

'x' such that    'x has some prop.'

Example: ① $\{\ x\ |\ 1 < x < 5\ \} = (1,5)$

② $\{\ x\ |\ 1 \leqslant x \leqslant 5\ \} = [1,5]$

③ $\{\ x\ |\ x \geqslant 0\ \} = [0, \infty)$

④ $\{\ x\ |\ x\ \text{is any real number}\ \}$

$\{\ x\ |\ x \in \mathbb{R}"\ \}$

$\hookrightarrow$ Belongs to

⊛ Interval notation: $(a,b) = \{\ x\ |\ a < x < b\ \} \longrightarrow$ Open Interval

$(a,b] = \{\ x\ |\ a < x \leqslant b\ \}$

$[a,b) = \{\ x\ |\ a \leqslant x < b\ \}$

$[a,b] = \{\ x\ |\ a \leqslant x \leqslant b\ \} \rightarrow$ Closed Interval.

⊛ Evaluating Functions: If $f(x) =$ some formula of $x$, then to evaluate, replace $x$ by the given point where you are asked to evaluate.

Eg: $f(x) = x^2 + 4x + 3$, evaluate at $x = 1, a, b+1$

$f(1) = 1^2 + 4 \cdot 1 + 3 = 1 + 4 + 3 = 8$

$f(a) = a^2 + 4a + 3$

$$f(b+1) = (b+1)^2 + 4(b+1) + 3$$
$$= (b^2 + 2b+1) + (4b+4) + 3$$
$$= b^2 + 6b + 8$$

$$(b+1)^2 = (b+1)(b+1) = b(b) + b(1) + 1(b) + 1(1)$$
$$= b^2 + b + b + 1$$
$$= b^2 + 2b + 1$$

✱ Finding Domain.

① $f(x) = \dfrac{3}{x^2 - 1}$

**We cannot divide by 0**

So, Domain $(f) = \left\{ x \mid x^2 - 1 \neq 0 \right\} = \left\{ x \mid x \neq \pm 1 \right\}$

② $f(x) = \dfrac{2x+5}{3x^2 + 4}$

$x^2 \geqslant 0$, for all real $x$

Since $3x^2 + 4 \geqslant 4$, for all real values of $x$.

So, Domain $(f) = (-\infty, \infty) = \mathbb{R}$

③ $f(x) = \sqrt{4 - 3x}$

↑
**Part under the square root symbol must be non-negetive.**

So, Domain $(f) = \left\{ x \mid 4 - 3x \geqslant 0 \right\}$
$$= \left\{ x \mid 4 \geqslant 3x \right\}$$
$$= \left\{ x \mid \frac{4}{3} \geqslant x \right\}$$
or
$$\left\{ x \mid x \leqslant \frac{4}{3} \right\}$$

⊛ Function Notation:

— by table

— by graph

| Hours after Midnight | Temperature (°F) | Hours after Midnight | Temperature (°F) |
|---|---|---|---|
| 0 | 58 | 12 | 84 |
| 1 | 54 | 13 | 85 |
| 2 | 53 | 14 | 85 |
| 3 | 52 | 15 | 83 |
| 4 | 52 | 16 | 82 |
| 5 | 55 | 17 | 80 |
| 6 | 60 | 18 | 77 |
| 7 | 64 | 19 | 74 |
| 8 | 72 | 20 | 69 |
| 9 | 75 | 21 | 65 |
| 10 | 78 | 22 | 60 |
| 11 | 80 | 23 | 58 |

Table 1.1 Temperature as a Function of Time of Day
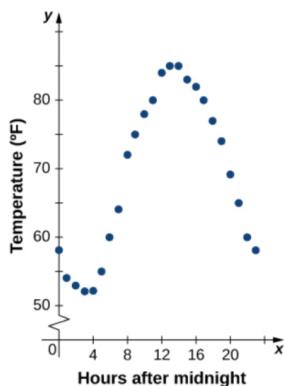
Figure 1.6   The graph of the data from Table 1.1 shows temperature as a function of time.

— by formula.

$$f(x) = \sqrt{x+3} + 1$$

Practice : Find the $\begin{cases} \text{domain of } f. \\ \text{zeros of } f \text{ (if any).} \end{cases}$
& try sketching it.

⊛ Piecewise-defined function.
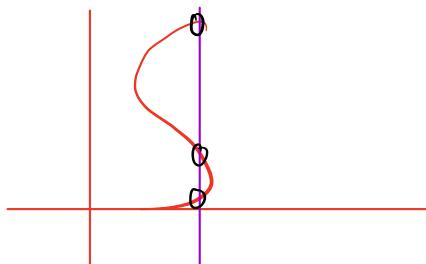
$$f(x) = \begin{cases} \text{Rule 1 for } x, & \text{when } x \text{ is in some set } A_1 \\ \text{Rule 2 for } x, & \text{when } x \text{ is in some other set } A_2 \end{cases}$$

& so on.

Eg:   $f(x) = \begin{cases} -x, & \text{when } x < 4 \\ 0, & \text{when } x = 4 \\ x^2, & \text{when } x > 4 \end{cases}$

Try to sketch.

⊛ Vertical Line Test: Given a function $f$, every vertical line that may be drawn intersects the graph of $f$, no more than once.

If it intersects more than once, then the set of points doesn't represent a function.



⊛ Zeros or x-intercepts & y-intercepts of a function.

$f(x) = -x + 2$ ⤳ only one zero at $x = 2$.
⤳ y-intercept is given by $(0, f(0))$
ie, $(0, 2)$

Eg. Let $f(x) = \sqrt{x^2 - 1}$

Zero or x-intercept: We need to find such $x$ for which $f(x) = 0$.

Note: $\sqrt{x} = 0$ is only possible if $x = 0$ & $x^2 = a^2 \Rightarrow x = \pm a$

Using this note we get, $f(x) = 0 \Rightarrow \sqrt{x^2 - 1} = 0 \Rightarrow x^2 - 1 = 0$
$\Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

So, x-intercepts are $x = -1$ & $x = 1$.

y-intercept: we need to find $f(0)$.

Now, $f(0) = \sqrt{0^2 - 1} = \sqrt{0 - 1} = \sqrt{-1}$, DNE $\Rightarrow$ No y-intercept in this case.