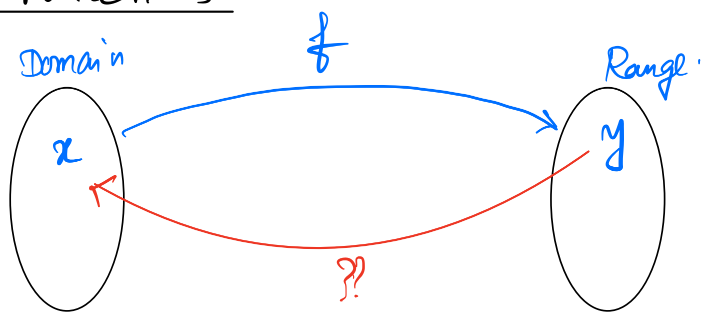
Inverse Functions



Given $f: \underset{\substack{\uparrow \\ \text{domain}}}{D} \longrightarrow \underset{\substack{\uparrow \\ \text{range}}}{R}$. If inverse function exists, then

it is denoted by $f^{-1}: R \longrightarrow D$ s.t. $\qquad f(x) = c = f(\bar{x})$

$$f^{-1}(f(x)) = x \ , \text{ for all } x \text{ in } D$$

$$\& \quad f(f^{-1}(y)) = y \ , \text{ for all } y \text{ in } R.$$

Note: ① $f: D \to R$ a constant function cannot have inverse function.

② f is one-one $\Rightarrow$ f possesses inverse function.

How to find inverse function?

① Solve the equation $y = f(x)$ for x.

② Interchange x & y & write $y = f^{-1}(x)$.

Example: $f(x) = 3x - 4 \Rightarrow f^{-1}(x) = \frac{1}{3}x + \frac{4}{3}$

Let $y = 3x - 4$. Then $y = 3x - 4$

$$\Rightarrow y + 4 = 3x \Rightarrow \boxed{x = \frac{1}{3}y + \frac{4}{3}}$$

Inverse Trigonometric Functions

$\sin^{-1}$: $D = \{x \mid -1 \leq x \leq 1\} \longrightarrow \{y \mid -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}\}$

$\sin^{-1}(x) = y \Leftrightarrow \sin(y) = x \,, -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}.$

$\cos^{-1}$: $D = \{x \mid -1 \leq x \leq 1\} \longrightarrow \{y \mid 0 \leq y \leq \pi\}$

$\cos^{-1}(x) = y \Leftrightarrow \cos(y) = x \,, \quad 0 \leq y \leq \pi.$

$\tan^{-1}$: $D = \{x \mid -\infty < x < \infty\} \longrightarrow (-\frac{\pi}{2}, \frac{\pi}{2})$

$\tan^{-1}(x) = y \Leftrightarrow \tan y = x \,, -\frac{\pi}{2} < y < \frac{\pi}{2}$

$\cot^{-1}$: $D = \{x \mid -\infty < x < \infty\} \longrightarrow (0, \pi)$

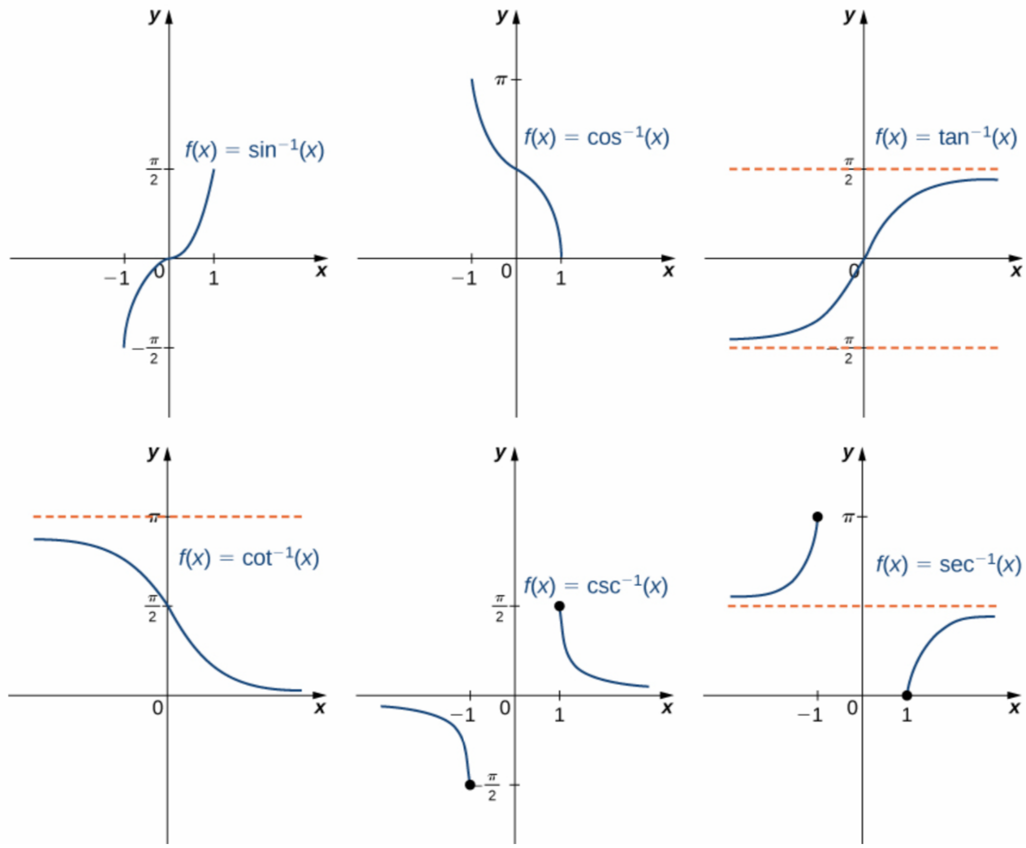
$\cot^{-1}(x) = y \Leftrightarrow \cot y = x \,, \quad 0 < y < \pi$

$\csc^{-1}$: $D = \{x \mid |x| \geq 1\} \longrightarrow \{y \mid -\frac{\pi}{2} \leq y \leq \frac{\pi}{2} \;\&\; y \neq 0\}$

$\csc^{-1}(x) = y \Leftrightarrow \csc(y) = x \,, -\frac{\pi}{2} \leq y \leq \frac{\pi}{2} \;\&\; y \neq 0$

$\sec^{-1}$: $D = \{x \mid |x| \geq 1\} \longrightarrow \{y \mid 0 \leq y \leq \pi \;\&\; y \neq \frac{\pi}{2}\}$

$\sec^{-1}(x) = y \Leftrightarrow \sec(y) = x \,, 0 \leq y \leq \pi \;\&\; y \neq \frac{\pi}{2}$



$f(x) = \sin^{-1}(x)$
$f(x) = \cos^{-1}(x)$
$f(x) = \tan^{-1}(x)$
$f(x) = \cot^{-1}(x)$
$f(x) = \csc^{-1}(x)$
$f(x) = \sec^{-1}(x)$