Exponential Functions

Lets start with an example:

Say you have \$ P_0 today in bank & you get 4% annual growth.

End of Year 1 $\rightsquigarrow$ \$ $(P_0 + P_0 \cdot (0.04)) = P_0(1.04)$

$$2 \rightsquigarrow \$(P_0(1.04) + P_0(1.04)(0.04))$$
$$= P_0(1.04)(1+0.04)$$
$$= P_0(1.04)(1.04)$$
$$= P_0(1.04)^2$$

$$3 \rightsquigarrow \$ P_0(1.04)^3$$

$$\vdots$$

$$r \rightsquigarrow \$ P_0(1.04)^r$$

More generally we get a special type of function

$$f(x) = b^x, \quad \text{where } b>0,\ b \neq 1$$

Exponential Function with base b & exponent x.

domain $(-\infty, \infty)$ & range $(0, \infty)$

Note: It is different from x^n, $n \geq 1$. Both grows, but exponential functions grows faster than power functions.

Most Common Applications:

1. Growth of money/Compound Interest
2. Growth of Bacterias/Cells.

Rules: ① $b^x \cdot b^y = b^{x+y}$ & $b^0 = 1$

② $\frac{b^x}{b^y} = b^{x-y}$

③ $b^{-y} = \frac{1}{b^y}$

④ $(b^x)^y = b^{xy}$

⑤ $(a \cdot b)^x = a^x \cdot b^x$

⑥ $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$.

Simplyfy : $\frac{(x^3 y^{-1})^2}{(xy^2)^{-2}}$

$$= \frac{(x^3)^2 (y^{-1})^2}{(x)^{-2} (y^2)^{-2}}$$

$$= \frac{x^6 \; y^{-2}}{x^{-2} \; y^{-4}}$$

$$= \frac{x^6}{x^{-2}} \cdot \frac{y^{-2}}{y^{-4}}$$

$$= x^{6-(-2)} \cdot y^{-2-(-4)}$$

$$= x^{6+2} y^{-2+4}$$

$$= x^8 y^2$$

A special Number 'e'

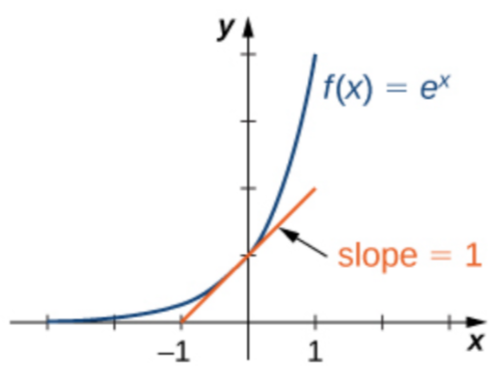
Observation :

m :	1	10	$10^2 = 100$	$10^3 = 1000$	$10^4 = 10000$	$10^5 = 100,000$	10^6
$\left(1+\frac{1}{m}\right)^m$:	2	2.5937	2.7048	2.71692	2.71815	2.718268	2.718280

If we take $m \longrightarrow \infty$, then $\left(1+\frac{1}{m}\right)^m \longrightarrow \underset{\substack{\parallel \\ e}}{\text{some fixed value}}$

$$e \approx 2.718282$$

$$\boxed{f(x) = e^x} \longrightarrow \text{Natural Exponential function}$$



Logarithmic Functions

Exponential functions are One-to-One
$\Downarrow$
Exponential functions possesses inverse.
Called Logarithmic Functions.

So, Domain of Log. Function is $(0, \infty)$
Range of Log. Function is $(-\infty, \infty)$

$$\log_b(x) = y \quad \text{if \& only if} \quad b^y = x.$$

When we use the base "e", ie, $\log_e$ function, we call it natural logarithm. ($\ln$)

Prop:

1. $\log_b(b^x) = x$ & $b^{\log_b(y)} = y$.
2. $\log_e(e^n) = \ln(e^n) = n$.
3. $\log_b(1) = 0$, for any base b
4. $\log_b(ac) = \log_b(a) + \log_b(c)$ ⇝ product rule
5. $\log_b\left(\frac{a}{c}\right) = \log_b(a) - \log_b(c)$ ⇝ Quotient rule.
6. $\log_b(a^r) = r \log_b(a)$ ⇝ power rule.
7. $a^x = b^{x \log_b a}$, for any real x.
8. $\log_a x = \dfrac{\log_b x}{\log_b a} = \dfrac{\ln x}{\ln a}$

$\Big\}\rightarrow$ Base change rules

Eg. Solve for x: $\ln(2x) - \ln(x^6) = 0$

$\ln(2x) - \ln(x^6) = 0$

$\Rightarrow \ln\left(\frac{2x}{x^6}\right) = 0$, quotient rule

$\Rightarrow \ln\left(\frac{2}{x^5}\right) = 0$

$\Rightarrow \frac{2}{x^5} = 1 \Rightarrow x^5 = 2 \Rightarrow x = \sqrt[5]{2}$